Math 184 Exam 1 SHOW ALL WORK Name [signature]

14 1. Solve each system (if possible). State the solution VERY clearly. If there is no solution, say so. You may use a calculator to put each augmented matrix in RREF.

a. $\begin{cases} 3x - y - 3z = 4 \\ 2x - y + 4z = 1 \\ 5x - 2y + z = 5 \end{cases}$

$\left[\begin{array}{ccc|c} 1 & 0 & -7 & 3 \\ 0 & 1 & -18 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\cancel{x} = 3+7t$

$y = 5+18t$

$t = t$

b. $\begin{cases} 2w + 3x - y - 3z = 4 \\ 5w - 2x - y + 4z = 1 \\ 7w + x - 2y + z = 0 \end{cases}$

$\left[\begin{array}{cccc|c} 1 & 0 & -\frac{5}{19} & \frac{6}{19} & 0 \\ 0 & 1 & -\frac{3}{19} & -\frac{23}{19} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$

no solution

10 | 0 0
9 | 0 9 39
9 | 0 9 4
2 | 3 9
6 | 5 7 0 1 6
5 | 3

10 2. If possible, find a so that: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 4 \\ 2 & a \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 17 & 10 \end{bmatrix}$. Show your work.

$\begin{bmatrix} 3+4 & 4+2a \\ 9+8 & 12+4a \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 17 & 10 \end{bmatrix} \Rightarrow$ $4+2a = 3$, $12+4a = 10$ $\Rightarrow$ $a = -\frac{1}{2}$

10 3. Find all values of the scalar a for which the matrix $\begin{bmatrix} 0 & 2 & a \\ 0 & a & 3 \\ 3 & 1 & 2 \end{bmatrix}$ is *not* invertible.

$\begin{vmatrix} 0 & 2 & a \\ 0 & a & 3 \\ 3 & 1 & 2 \end{vmatrix} = 3\begin{vmatrix} 2 & a \\ a & 3 \end{vmatrix} = 3(6-a^2) = 0$ if $a = \pm\sqrt{6}$

6 4. Assume A,B,C,D are nxn matrices, and A and C are invertible. Solve the following **matrix** equation for B, using steps appropriate for **matrices**:

ABC = D

$A^{-1}ABC = A^{-1}D$

$IBC = A^{-1}D$

$BC = A^{-1}D$

$BCC^{-1} = A^{-1}DC^{-1}$

$BI = A^{-1}DC^{-1}$

$\boxed{B = A^{-1}DC^{-1}}$

10 5. Recall that the trace of a matrix, denoted tr(A), is just the sum of the diagonal entries. Prove that for any scalar c, tr(cA) = c tr(A).

$$tr(cA) = \sum_{i=1}^{n} ent_{ii}(cA)$$

$$= \sum_{i=1}^{n} c \cdot ent_{ii}(A)$$

$$= c \cdot \sum_{i=1}^{n} ent_{ii}(A)$$

$$= c \cdot tr(A)$$

10

6. Determine if the set of all 3x1 vectors is a vector space under the standard operation of vector addition with scalar multiplication defined as follows: $k \cdot \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} kd \\ d+f \\ kf \end{bmatrix}$.

Justify your answer either by showing at least one axiom that fails and how it fails or else by showing the zero and additive inverse vectors.

axiom 8 requires that $1 \cdot \vec{v} = \vec{v}$

Put $1 \cdot \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 \cdot d \\ d+f \\ 1 \cdot f \end{bmatrix} = \begin{bmatrix} d \\ d+f \\ f \end{bmatrix} \neq \begin{bmatrix} d \\ e \\ f \end{bmatrix}$

So not a vector space

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7. Is the set of all symmetric matrices a *subspace* of M_{nxn}? Justify your answer.

① The sum of symmetric matrices is symmetric (Thm 1.14.1)

② If A is symmetric, cA is. (Thm 1.14.2)

So closed under ⊕ and ⊙ so yes

8. Give a basis for the set of *diagonal* 3x3 matrices.

$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

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9. Let the vector <a,b,c> be an element of the subspace of R^3 spanned by the set of vectors {<1,0,0>, <2,-1,3>,<1,2,-5>} . What are the conditions necessary for a,b,c? In other words, are there any restrictions on a,b,c? Answer VERY clearly.

Let $\langle a,b,c\rangle = c_1\langle 1,0,0\rangle + c_2\langle 2,-1,3\rangle + c_3\langle 1,2,-5\rangle$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & a \\ 0 & -1 & 2 & b \\ 0 & 3 & -5 & c \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & a \\ 0 & 1 & -2 & -b \\ 0 & 0 & 1 & c+3b \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & a-c-3b \\ 0 & 1 & 0 & 2c+5b \\ 0 & 0 & 1 & c+3b \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & a-3c-13b \\ 0 & 1 & 0 & 2c+5b \\ 0 & 0 & 1 & c+3b \end{array}\right]$$

So: $c_1 = c+3b$

$c_2 = 5b+2c$

$c_3 = a-3c-13b$

These values are defined for all a, b, c

so no restrictions

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10. Are the matrices $\left\{\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}, \begin{bmatrix}1 & 1\\3 & 2\end{bmatrix}, \begin{bmatrix}0 & 2\\1 & 0\end{bmatrix}, \begin{bmatrix}3 & 6\\10 & 8\end{bmatrix}\right\}$ linearly independent? Explain, using the definition of linear independence.

Let $c_1\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} + c_2\begin{bmatrix}1 & 1\\3 & 2\end{bmatrix} + c_3\begin{bmatrix}0 & 2\\1 & 0\end{bmatrix} + c_4\begin{bmatrix}3 & 6\\10 & 8\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 3 & 0 \\ 2 & 1 & 2 & 6 & 0 \\ 3 & 3 & 1 & 10 & 0 \\ 4 & 2 & 0 & 8 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow c_3 = -c_4$$

There are nontrivial solutions to the system, so $\boxed{\text{no}}$