FACTORING POLYNOMIALS

1) **<u>Common Factor –</u>** If each term has a common factor, <u>always</u> factor it out before proceeding with factoring.

   $$3x^4 + 6x^3 - 12x^2 = 3x^2(x^2 + 2x - 4)$$

   Sometimes it is helpful to factor out a -1. For example, $\frac{x-3}{3-x} = \frac{x-3}{-(x-3)} = -1$

2) **<u>Trial and Error</u>**

   $$x^2 + 2x - 3 = (x + ?)(x - ?) = (x + 3)(x - 1)$$

3) **<u>Grouping</u>**

   $$x^3 - x^2 + x - 1 = \underline{x^3 - x^2} + \underline{x - 1}$$

   $$= x^2(x - 1) + 1(x - 1)$$

   $$= (x - 1)(x^2 + 1)$$

4) **<u>Diamond Method of Factoring</u>** – *See Handout on Diamond Method of Factoring*

5) **<u>Special Forms to Recognize</u>**

   a) **<u>Sum of Squares</u>**

      $A^2 + B^2$ **(cannot be factored)**

   b) **<u>Difference of Squares</u>**

      $A^2 - B^2 = ?^2 - ?^2 = A^2 - B^2 = (A - B)(A + B)$

      Example: $x^2 - 9 = ?^2 - ?^2 = x^2 - 3^2 = (x - 3)(x + 3)$

      Example: $4x^2 - 81y^2 = ?^2 - ?^2 = (2x)^2 - (9y)^2 = (2x - 9y)(2x + 9y)$

   c) **<u>Sum or Difference of Cubes</u>**

      Sum: $A^3 + B^3 = ?^3 + ?^3 = A^3 + B^3 = (A + B)(A^2 - AB + B^2)$
      Note: $(A^2 - AB + B^2)$ **cannot be factored any further**

      Diff: $A^3 - B^3 = ?^3 - ?^3 = A^3 - B^3 = (A - B)(A^2 + AB + B^2)$
      Note: $(A^2 + AB + B^2)$ **cannot be factored any further**

      Example: $x^3 + 8 = ?^3 + ?^3 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 2^2) = (x + 2)(x^2 - 2x + 4)$

      Example: $27x^3 - 1 = ?^3 - ?^3 = (3x)^3 - 1^3 = (3x - 1)\big((3x)^2 + 3x + (1)^2\big)$
      $= (3x - 1)(9x^2 + 3x + 1)$

d) Perfect Squares

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

Example: $x^2 - 8x + 16 = (x - 4)^2$

e) Perfect Cubes

$A^3 + 3A^2B + 3AB^2 + B^3 = (A + B)^3$

$A^3 - 3A^2B + 3AB^2 - B^3 = (A - B)^3$

Example: $x^3 - 9x^2 + 27x - 27 = (x - 3)^3$

6) Completing the Square

If $x^2 + bx$ is a binomial, then by add $\left(\frac{b}{2}\right)^2$ to create a perfect square trinomial.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example: What term should be added to $x^2 + 8x$ to create a perfect square trinomial?

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = x^2 + 8x + 4^2 = x^2 + 8x + 16 = (x + 4)^2$$

Example: Solve by completing the square: $x^2 - 6x + 4 = 0$

First, subtract 4 from each side of the equation to isolate the binomial $x^2 - 6x$:

$x^2 - 6x + 4 = 0$
$x^2 - 6x = -4$

Next, add $\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$ to each side of the equation:

$x^2 - 6x + 9 = -4 + 9$

$(x - 3)^2 = 5$

By taking the square root of each side of the equation:

$$x - 3 = +\sqrt{5} \text{ or } x - 3 = -\sqrt{5}$$

Therefore, $x = 3 + \sqrt{5}$ **or** $x = 3 - \sqrt{5}$