

ASYMPTOTES

Rational Functions

A *rational function* is a function f that is a quotient of two polynomials, that is,

$$f(x) = \frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials with no common factor other than ± 1 and $Q(x)$ is not the zero polynomial. The domain of f consists of all inputs x for which $Q(x) \neq 0$.

Vertical Asymptotes (VA)

These asymptotes occur when the denominator $Q(x) = 0$. They have the form $x = a$.

Example: $f(x) = \frac{2x-1}{x^2-1} = \frac{2x-1}{(x-1)(x+1)}$

The denominator is 0 when $x = 1$ and $x = -1$. The vertical lines $x = 1$ and $x = -1$ are the vertical asymptotes.

Horizontal Asymptotes (HA)

These asymptotes occur when the degree of the numerator is less than or equal to the degree of the denominator.

- If the $\deg(\text{num}) < \deg(\text{den})$, then the horizontal asymptote is $y = 0$ (the x -axis)

Example: $f(x) = \frac{2x-1}{x^2-1}$ $\deg(\text{num}) = 1$ and $\deg(\text{den}) = 2$

So the horizontal asymptote is $y = 0$

- If the $\deg(\text{num}) = \deg(\text{den})$, then the asymptote is $y = \frac{a}{b}$, where a is the leading coefficient of the numerator and b is the leading coefficient of the denominator.

Example: $f(x) = \frac{2x^2-1}{x^2-1}$ $\deg(\text{num}) = 2$ and $\deg(\text{den}) = 2$

So the horizontal asymptote is $y = \frac{2}{1} = 2$

Slant or Oblique Asymptotes (SA)

These asymptotes occur when the degree of the numerator is one larger than the degree of the denominator. If this occurs you will have to do polynomial division and the quotient (the answer) is your slant asymptote.

Example: $f(x) = \frac{2x^2-1}{x-1}$ $\deg(\text{num}) = 2$ and $\deg(\text{den}) = 1$

$$\begin{array}{r} 2x + 2 \\ x - 1 \overline{) 2x^2 - 1} \\ \underline{2x^2 - 2x} \\ 2x - 1 \\ \underline{2x - 2} \\ 1 \end{array} \quad \begin{array}{l} \text{The equation of the slant asymptote is } y = 2x + 2 \\ \text{(Note the remainder does not affect this.)} \end{array}$$

NOTE: You will never have both a SA and a HA on the same graph. Graphs can cross HA and SA, but will never cross VA.

Recommended Points to Plot

y-intercept ($x = 0$) if one exists

x-intercepts ($y = 0$) if they exist

two points close to each vertical asymptote (one on each side)

a point out towards each "end" of the graph. For example ± 10 or ± 100

NOTE: *You may need or want to plot some other points as well.*

Example: $f(x) = \frac{2x-1}{x^2-1}$

VA: $x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$ ← these are your VA's

HA: since $\text{deg}(\text{num}) = 1 < \text{deg}(\text{den}) = 2$; then $y = 0$ is the HA.

SA: since you have a HA, you do not have a SA.

y-intercept: $x = 0$

$$y = \frac{2(0)-1}{(0)^2-1} = \frac{-1}{-1} = 1$$

Therefore the y-intercept is (0,1)

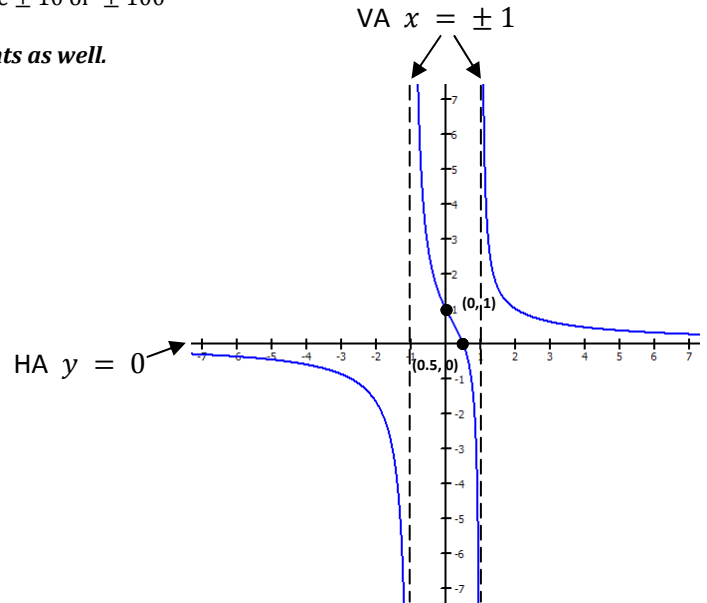
x-intercept: $y = 0$

$$0 = \frac{2x-1}{x^2-1}$$

$$0 = 2x - 1$$

$$x = \frac{1}{2} = 0.5$$

Thus, the x-intercept is (0.5, 0)



X	Y
0	1
1	Undefined
-1	Undefined
0.5	0
1.25	≈ 2.66667
-1.25	≈ -6.22222
6	≈ 0.314286
-6	≈ -0.371429

Points close to VA:

$x = \pm 1.25$

$$y = \frac{2(1.25)-1}{(1.25)^2-1} = \frac{1.5}{0.5625} \approx 2.66667$$

$$y = \frac{2(-1.25)-1}{(-1.25)^2-1} = \frac{-3.5}{0.5625} \approx -6.22222$$

Points close to HA:

$x = \pm 6$

$$y = \frac{2(6)-1}{(6)^2-1} = \frac{11}{35} \approx 0.314286$$

$$y = \frac{2(-6)-1}{(-6)^2-1} = \frac{-13}{35} \approx -0.371429$$