Using the Binomial Distribution

The following problem will be used to practice working with the binomial distribution using the calculator:

A smoothie shop sells on average 80 strawberry, 47 raspberry, and 23 banana smoothies on any given day.

Problems:

a.) Out of the first 20 smoothies sold, what is the probability that exactly 15 were strawberry?

b.) Out of the first 20 smoothies sold, what is the probability of selling fewer than 10 strawberry smoothies?

c.) Out of the first 20 smoothies sold, what is the probability that more than 15 strawberry smoothies will be sold?

*Think first: should the probabilities for each be relatively high, low, etc.? Does your answer make sense in the context of the problem?

The questions being asked indicate that this is a binomial distribution. We can see that because the experiment has a fixed number of independent trials and there are only two possible outcomes that can be defined as a success or a failure. The probability of success is the same for each trial. The formula for a binomial distribution is:

$$P(x) = {}_nC_x\ p^x\ q^{n-x}$$

This formula is the probability of exactly *x* successes in *n* trials in a binomial experiment.

In this problem, 80 out of the 150 smoothies sold are strawberry, so the probability of success is

$p = \dfrac{80}{150}$ which equals 0.533. This means that about half of the smoothies sold are typically strawberry.

a.) P(15)

Think: Since about half of the smoothies sold are typically strawberry, in looking at the first 20 smoothies sold, we would expect about 10 of those to be strawberry; that is, the probability of exactly 10 of the 20 smoothies being strawberry should be relatively high. However, the question asks about exactly 15 smoothies being strawberry. Since 15 is not relatively close to 10, we should expect the probability to be small.
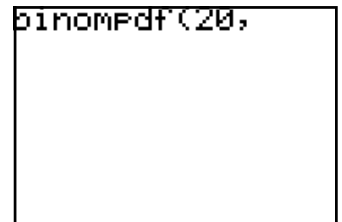
We want P(15), and we can do it in the calculator using the *binompdf* function.

First, input:

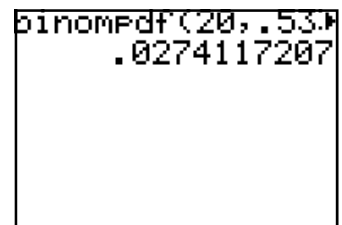   *2nd > **DISTR** > scroll down to **A: binompdf(** > ENTER > 20, 0.533, 15)*

(Screen 1)

ENTER

(Screen 2)

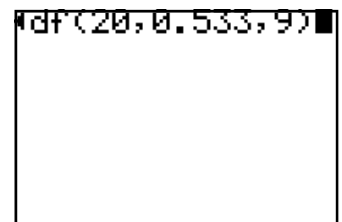Out of the first 20 smoothies sold, there is a 0.027 (or about 3%) probability that 15 of those are strawberry.

b.) P(x < 10)

Selling fewer than 10 strawberry smoothies means P(x < 10) = P(x ≤ 9).

Now use the *binomcdf* function in the calculator by doing this:

    *2nd > **DISTR** > scroll down to **B: binomcdf(** > ENTER > 20, 0.533, 9)*

(Screen 3)

Screen 1

Screen 2

Screen 3

ENTER

(Screen 4)

Here, we get a probability of 0.300 (or about 30%) for selling fewer than 10 strawberry smoothies out of the first 20 smoothies sold.

```
binomcdf(20,.53.▶
        .3008785408
```

c.) P(x > 15)

Here we want the probability of selling more than 15 strawberry smoothies out of the first 20 smoothies sold, x > 15. The complement is x ≤ 15, which means that we have to subtract P(x ≤ 15) from 1.

$$P(x > 15) = 1 - P(x \le 15)$$

In the calculator, input:

  *1 > - > 2nd > **DISTR** > scroll down to **B: binomcdf(** > **ENTER** > 20, 0.533, 15)*

(Screen 5)

```
1-binomcdf(20,
```

ENTER

(Screen 6)

Out of the first 20 smoothies sold, the probability that more than 15 strawberry smoothies will be sold is 0.013 (or about 1%). This makes sense because we expect the probability to be small. We figured out that about half of the smoothies sold in a day are strawberry, so we expect to sell about 10 strawberry smoothies out of the first 20 smoothies sold. Selling more than 15 is not very likely.

```
1-binomcdf(20,.▶
        .0129651932
```

NOTE: Anytime when using the calculator, how do you know whether to use the *pdf* function or the *cdf* function?

Use the **cdf** function when you are looking for a sum over a range of values from 0 up to and including a specific number (less than or equal to).

> Ex. P(x ≤ 10)

Use the **pdf** function when you are looking for a single value of x (equality).

> Ex. P(x = 100) = P(100)