PROPERTIES OF LOGARITHMS

Definition:

For $x, b > 0, b \neq 1$ $\quad \log_b x = y \quad \Leftrightarrow \quad b^y = x$

Natural Logarithm $\quad \ln x = \log_e x$

Common Logarithm $\quad \log x = \log_{10} x$

Property Name	Property	Example
One-to-one	$\log_b y = \log_b x \quad \Leftrightarrow \quad x = y,$ for $b > 0, b \neq 1$	$\log_{10} x = \log_{10} 8$ $x = 8$
Property of One	$\log_b 1 = 0$	$\log_5 1 = 0$
Multiplication Property	$\log_b(xy) = \log_b x + \log_b y$	$\log_2(5x) = \log_2 5 + \log_2 x$
Division Property	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_6\left(\frac{x}{7}\right) = \log_6 x - \log_6 7$
Power Property	$\log_b x^r = r \log_b x$	$\log_3 x^5 = 5 \log_3 x$
Inverse Property	$b^{\log_b x} = x$ and $\log_b b^x = x$ Therefore: $\log_b b = 1$ $\ln e = 1$ $\log 10 = 1$	$4^{\log_4 6} = 6$ $\log_4 4^6 = 6$ If $\ln\frac{x+2}{4x+3} = \ln\frac{1}{x}$, then $e^{\ln\left(\frac{x+2}{4x+3}\right)} = e^{\ln\left(\frac{1}{x}\right)}$ and $\frac{x+2}{4x+3} = \frac{1}{x}$
Change of Base	$\log_b x = \frac{\log_a x}{\log_a b}$	$\log_8 11 = \frac{\log_5 11}{\log_5 8} = \frac{\log 11}{\log 8} = \frac{\ln 11}{\ln 8}$

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Examples

1. Solve by using the Definition:

$$\log_4 64 = y \Leftrightarrow 4^y = 64$$

$$4^3 = 64$$

Therefore: y = 3 and $\log_4 64 = 3$

2. Simplify by using the Multiplication Property and Definition:

$$\log_4 2 + \log_4 32 = \log_4(2 \cdot 32)$$

$$= \log_4 64$$

$$= 3$$

3. Simplify by using the Power Property and Multiplication Property:

$$2\ln x + \ln(x+1) = \ln x^2 + \ln(x+1)$$

$$= \ln[x^2(x+1)]$$

$$= \ln(x^3 + x^2)$$

4. Expand by using the Multiplication Property and Power Property:

$$\log\left(x^2\sqrt{y}\right) = \log\left(x^2 y^{\frac{1}{2}}\right)$$

$$= \log x^2 + \log y^{\frac{1}{2}}$$

$$= 2\log x + \frac{1}{2}\log y$$

5. Solve by using the Division Property:

$$\ln(x+2) - \ln(4x+3) = \ln\left(\frac{1}{x}\right)$$

$$\ln\left(\frac{x+2}{4x+3}\right) = \ln\left(\frac{1}{x}\right)$$

(One-to-one or Inverse) $\frac{x+2}{4x+3} = \frac{1}{x}$

$$x(x+2) = 4x+3$$

$$x^2 + 2x = 4x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

Therefore: $x = 3$ and $x = -1$

<u>Always check proposed solutions</u> of a logarithmic equation in the original equation. Exclude from the solution set any proposed solution that produces the log of a negative number or the log of 0.

$x = -1$ does not work since it produces the log of a negative number. Therefore, the solution is: $x = 3$

6. Solve by using the Inverse Property:

$$6e^{12x} = 18$$

$$e^{12x} = \frac{18}{6}$$

$$\ln e^{12x} = \ln\left(\frac{18}{6}\right)$$

$$12x = \ln(3)$$

$$x = \frac{\ln(3)}{12} \approx .092$$