The Diamond Method of Factoring a Quadratic Equation

Important:

- Remember that the first step in any factoring is to look at each term and factor out the greatest common factor. For example: $3x^2 + 6x + 12 = 3(x^2 + 2x + 4)$ AND $5x^2 + 10x = 5x(x + 2)$
- If the leading coefficient is negative, always factor out the negative. For example:
  $-2x^2 - x + 1 = -1(2x^2 + x - 1) = -(2x^2 + x - 1)$

Using the Diamond Method:

Example 1

Factor **$2x^2 + 11x + 15$** using the Diamond Method.

Step 1: Multiply the coefficient of the x^2 term **(+2)** and the constant **(+15)** and place this product **(+30)** in the top quarter of a large "X."

Step 2: Place the coefficient of the middle term in the bottom quarter of the "X." **(+11)**

Step 3: List all factors of the number in the top quarter of the "X."

+30	
(+1)(+30)	**(-1)(-30)**
(+2)(+15)	**(-2)(-15)**
(+3)(+10)	**(-3)(-10)**
⟶ **(+5)(+6)**	**(-5)(-6)**

Step 4: Identify the two factors whose sum gives the number in the bottom quarter of the "x." **(5 · 6 = 30 and 5 + 6 = 11)** and place these factors in the left and right quarters of the "X" (order is not important).

Step 5: Break the middle term of the original trinomial into the sum of two terms formed using the right and left quarters of the "X." That is, write the first term of the original equation, **$2x^2$** , then write **11x** as **+ 5x + 6x** (the numbers from the "X"), and finally write the last term of the original equation, **+15** , to get the following 4-term polynomial:

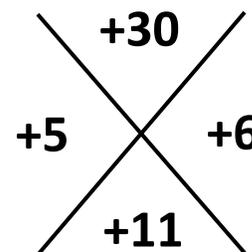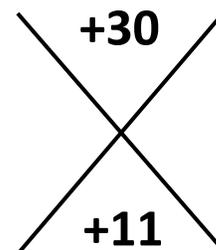$$2x^2 + 11x + 15 = 2x^2 + 5x + 6x + 15$$

Step 6: Factor by Grouping:

Group the first two terms together and the last two terms together. $\underline{2x^2 + 5x} + \underline{6x + 15}$

Factor out common factors from each group. $x(2x + 5) + 3(2x + 5)$

Factor out the common binomial factor. $\boxed{(2x + 5)(x + 3)}$
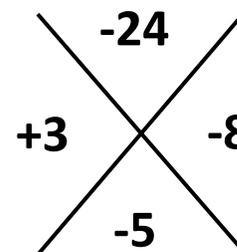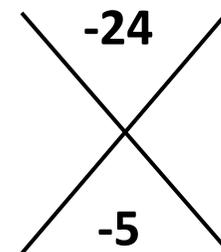
Example 2

Factor $12x^2 - 5x - 2$ using the Diamond Method.

Step 1: Multiply the coefficient of the x^2 term **(+12)** and the constant **(-2)** and place this product **(-24)** in the top quarter of a large "X."

Step 2: Place the coefficient of the middle term in the bottom of the "X." **(-5)**

Step 3: List all factors of **-24**:

-24	
(+1)(-24)	(-1)(+24)
(+2)(-12)	(-2)(+12)
⟶ (+3)(-8)	(-3)(+8)
(+4)(-6)	(-4)(+6)

Step 4: Identify the factors whose sum is **−5**: **(+3 · -8 = -24 and 3 - 8 = -5)** and place them in the left and right quarters of the "X" (order is not important).

Step 5: Break the middle term of the original trinomial into the sum of two terms formed using the right and left quarters of the "X." $12x^2 - 5x - 2 = 12x^2 + 3x - 8x - 2$

Step 6: Factor by Grouping:

Group the first two terms together and the last two terms together. $\underline{12x^2 + 3x} \ \underline{- 8x - 2}$

Factor out common factors from each group. (Factor out negative.) $3x(4x + 1) - 2(4x + 1)$

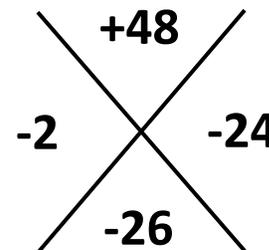Factor out the common binomial factor. $\boxed{(4x + 1)(3x - 2)}$

Example 3

Factor $16x^2 - 26x + 3$ using the Diamond Method.

Step 1: **(+48)**   Step 2: **(-26)**

Step 3: List all factors of **48**:

+48	
(+1)(+48)	(-1)(-48)
(+2)(+24)	(-2)(-24) ⟵
(+3)(+16)	(-3)(-16)
(+4)(+12)	(-4)(-12)
(+6)(+8)	(-6)(-8)

Step 4: Identify the factors whose sum is **-26**:   **- 2 - 24 = - 26**

Step 5: $16x^2 - 26x + 3 = 16x^2 - 2x - 24x + 3$

Step 6: Factor by Grouping:

Group the first two terms together and the last two terms together. $\underline{16x^2 - 2x} \ \underline{- 24x + 3}$

Factor out common factors from each group. (Factor out negative.) $2x(8x - 1) - 3(8x - 1)$

Factor out the common binomial factor. $\boxed{(8x - 1)(2x - 3)}$